AKADEMIA GÓRNICZO-HUTNICZA
IM. STANISŁAWA STASZICA W KRAKOWIE

UNIWERSYTET ZIELONOGÓRSKI

Deadlock-Prone Circuits in S^3PR Petri Nets

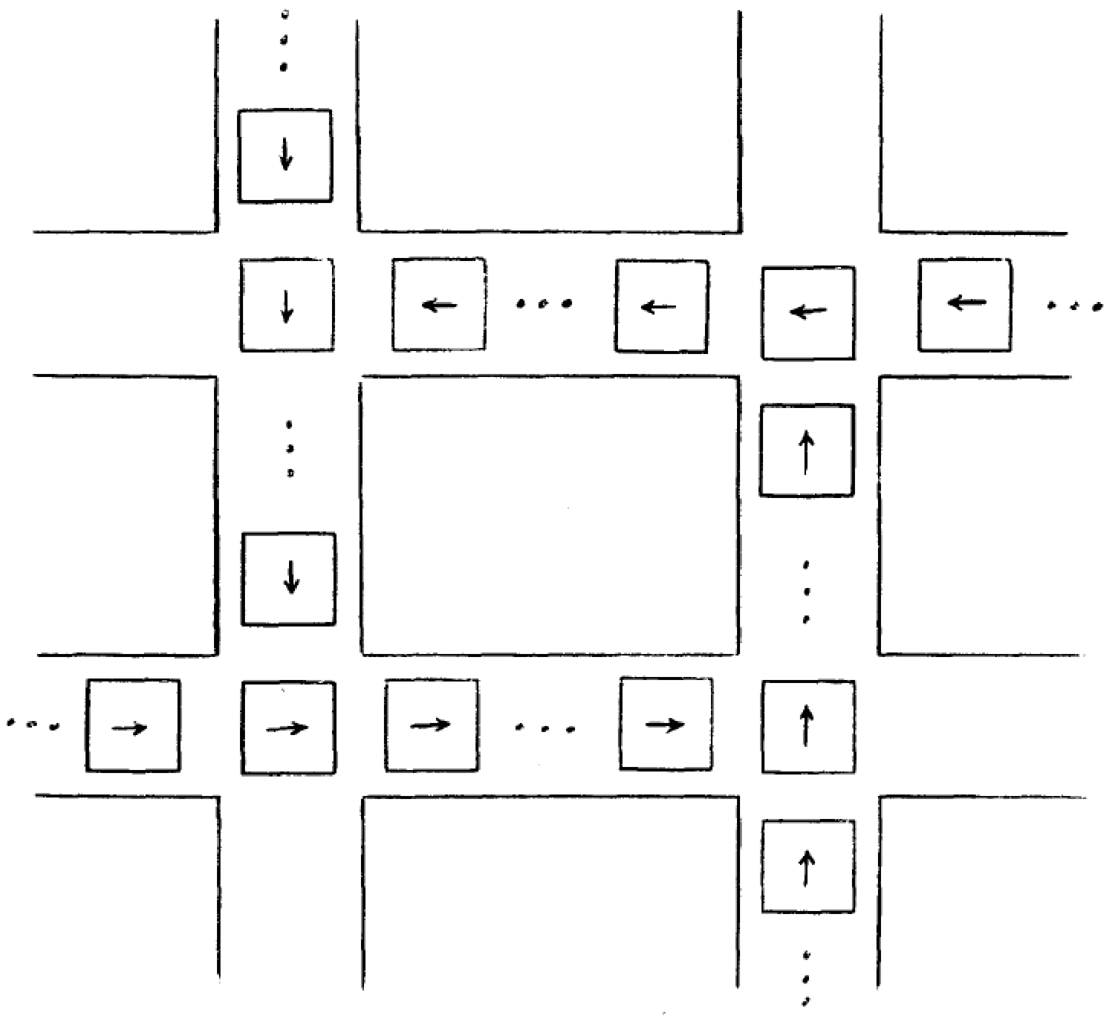
Andrei Karatkevich, AGH University
Iwona Grobelna, University of Zielona Góra

IEEE EUROCON 2025, Gdynia, June, 4-6, 2025

Motivation

- Deadlocks are an essential problem in parallel systems
- Deadlock detection is a hard task
- There are classical deadlock conditions (Coffman, 1971)
- The conditions are related to the limited class of situations
- Our motivation is to consider the conditions for more general situations
- We use the S^3PR Petri nets as the model

Deadlock



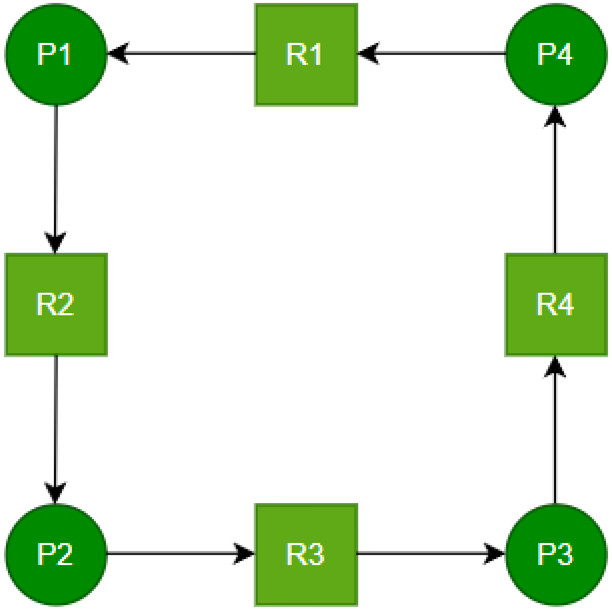
Deadlock

- A state of a parallel program or a parallel system, in which a set of processes cannot proceed because each of them waits for an operation to be executed from another process of this set
- A state of a program or a parallel system which cannot change
- Global and local deadlocks:
  - A global deadlock – the whole system is blocked
  - A local deadlock – some of the system processes block each other
- Sometimes liveness is considered, a wider notion: lack of the dead fragments of the system

Conditions for deadlock (on resources) - the Coffman conditions

- *Mutual exclusion*: only one process at a time may use each resource.
- *Resource holding:* processes hold resources already allocated to them while waiting for additional resources.
- *No preemption*: Resources cannot be forcibly removed from the processes holding them
- ***Circular wait***: A circular chain of processes exists, such that each process holds one or more resources that are being requested by the next task in the chain

Circular wait: a simple example



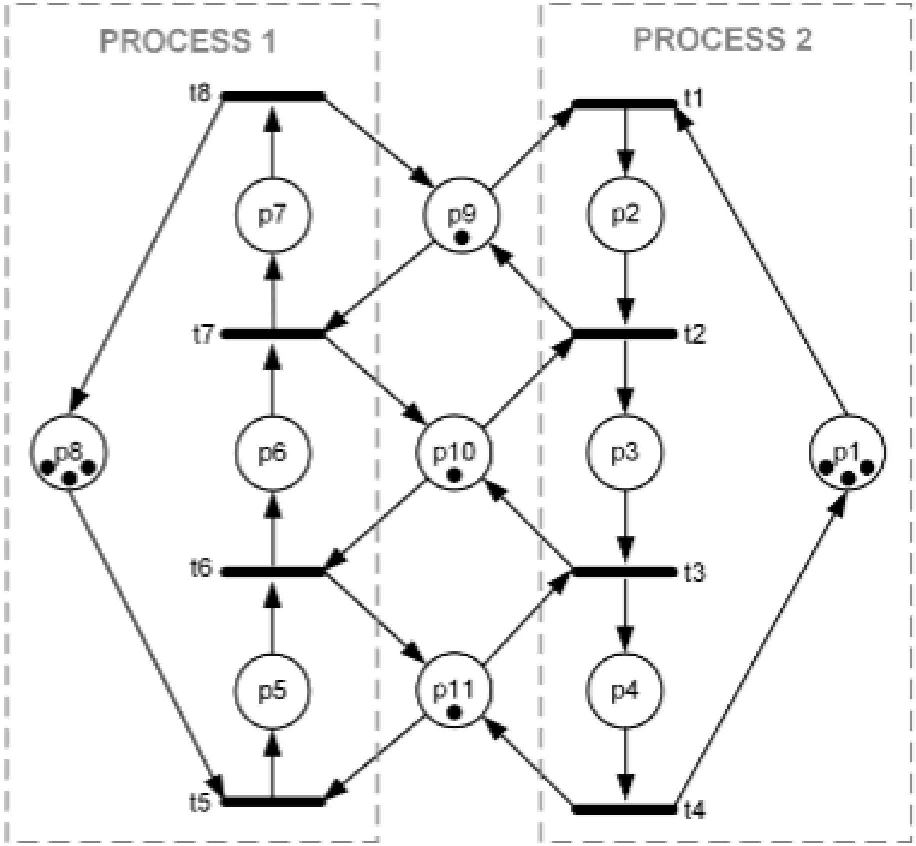
Circular wait

- A necessary condition for a deadlock
- A sufficient condition, when other 3 conditions are satisfied
- Detecting cycles in the resource flow graphs is a popular method for deadlock detection
- Some of the problems not covered with the classical approach:
  - When there are several instances of a resource (no mutual exclusion)
  - When there are alternative ways for evolution of a process

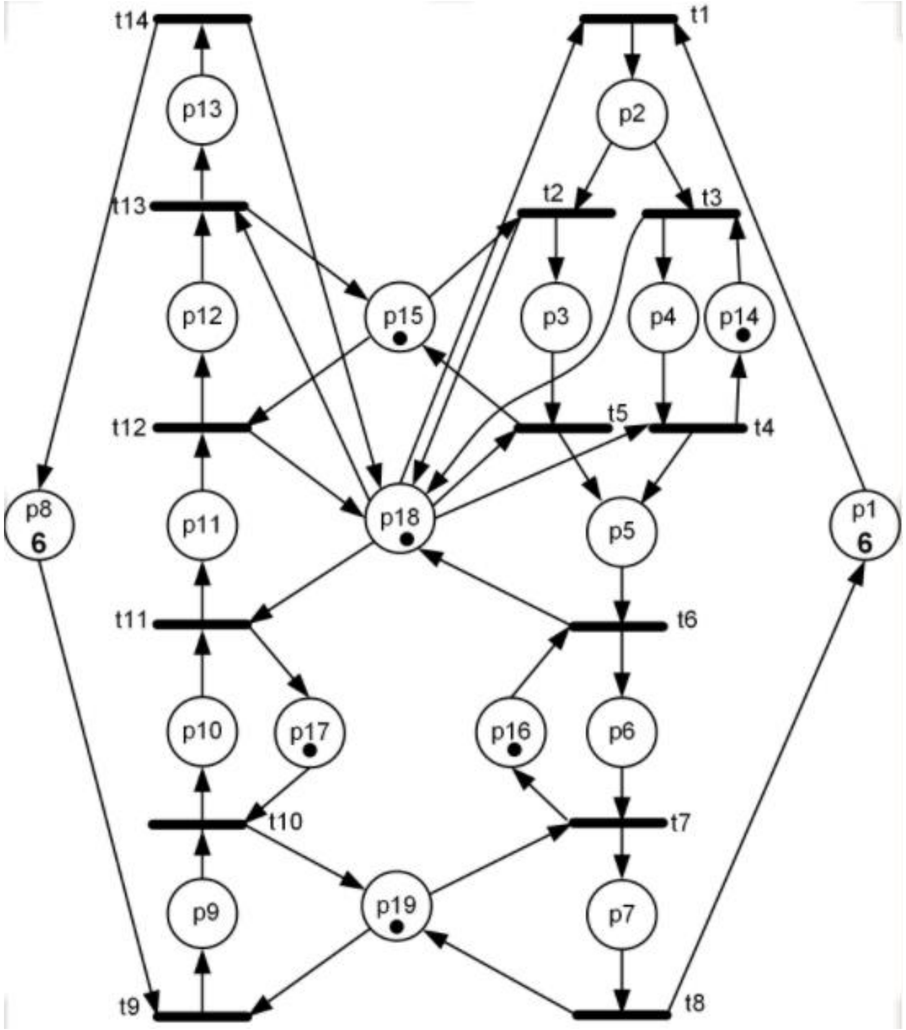
S^3PR Petri nets: the model we use

- A System of Simple Sequential Processes with Resources
- Introduced in:
  - J. Ezpeleta, J. M. Colom and J. Martinez, ”A Petri net based deadlock prevention policy for flexible manufacturing systems”, IEEE Trans. Robot. and Autom., vol. 11, no. 2, Apr. 1995.
- The most popular Petri net model of flexible manufacturing systems
- The system consists of the sequential processes with shared resources

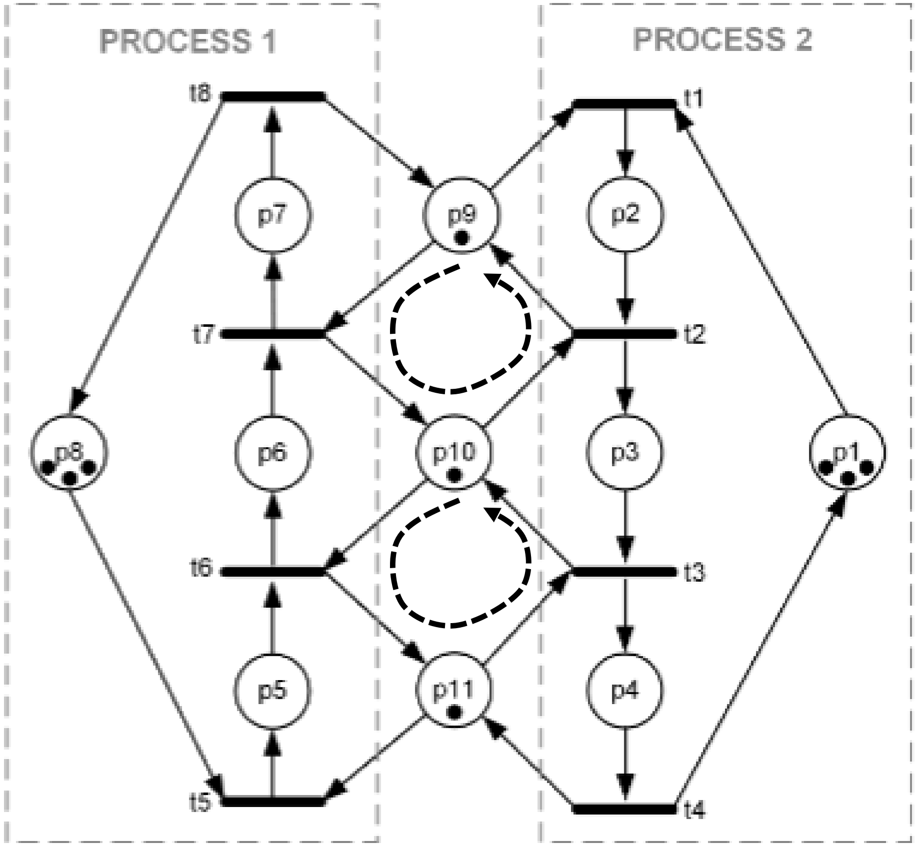
S^3PR: an example



S^{3}PR: an example

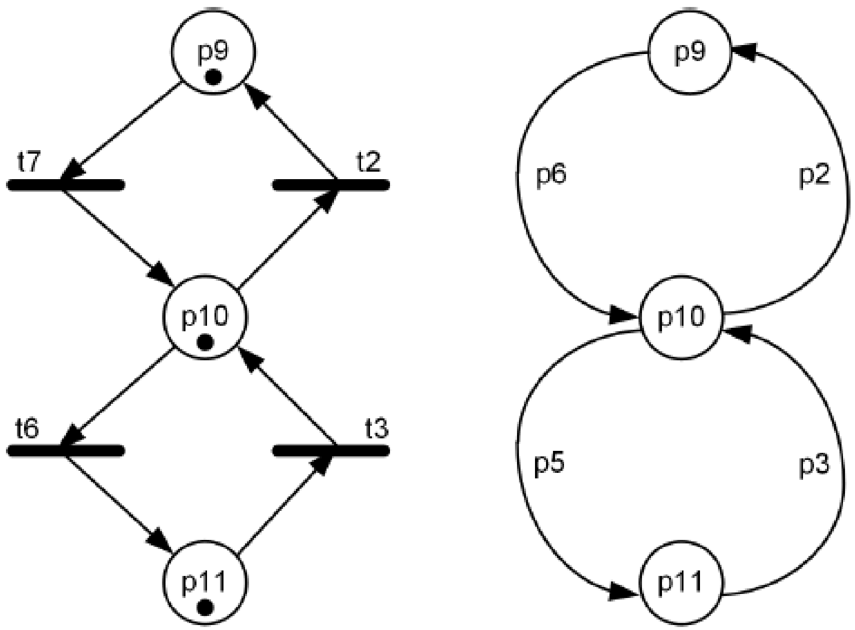


Resource flow graphs in S^3PR nets



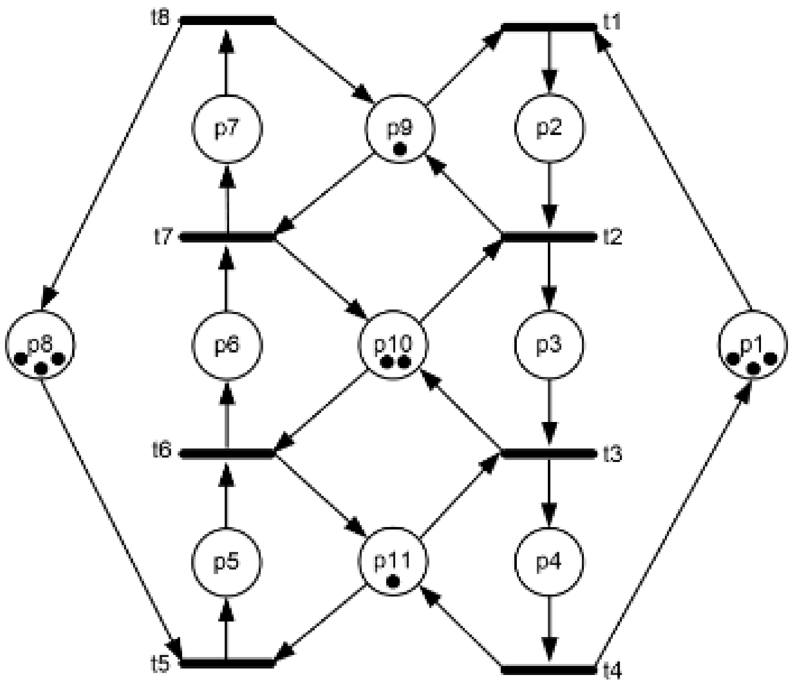
- Can be directly obtained from the net graphs – consist of the resource places and the transitions
- At least one cycle in the wait-for graph corresponds to every deadlock

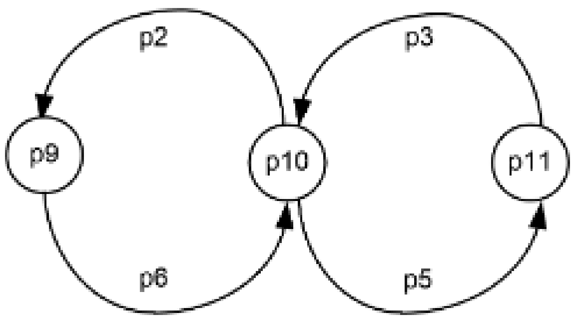
Resource flow graphs in S^3PR nets



- Can be directly obtained from the net graphs – consist of the resource places and the transitions
- At least one cycle in the wait-for graph corresponds to every deadlock

What if there are more resources?





Some kinds of markings

- An ***acceptable initial marking***:
  - One or more tokens in every idle place
  - One or more tokens in every resource place
  - No tokens in the activity places
- A ***possible marking*** (for given initial marking):
  - Sum of the tokens in the idle and activity places of a sequential process is the same as in the initial marking
  - Sum of the tokens in a resource place and its holders is the same as in the initial marking
- A ***reachable marking***: as for general Petri nets

Some formal results

Proposition 1. For every circle *C* in the resource flow graph of a conflict-free S^3PR *N*, there exist:

(1) an acceptable initial marking of *N*, for which there is a possible marking being a deadlock corresponding to *C*, and

(2) an acceptable initial marking of *N*, for which there is no possible marking being such a deadlock.

Some formal results

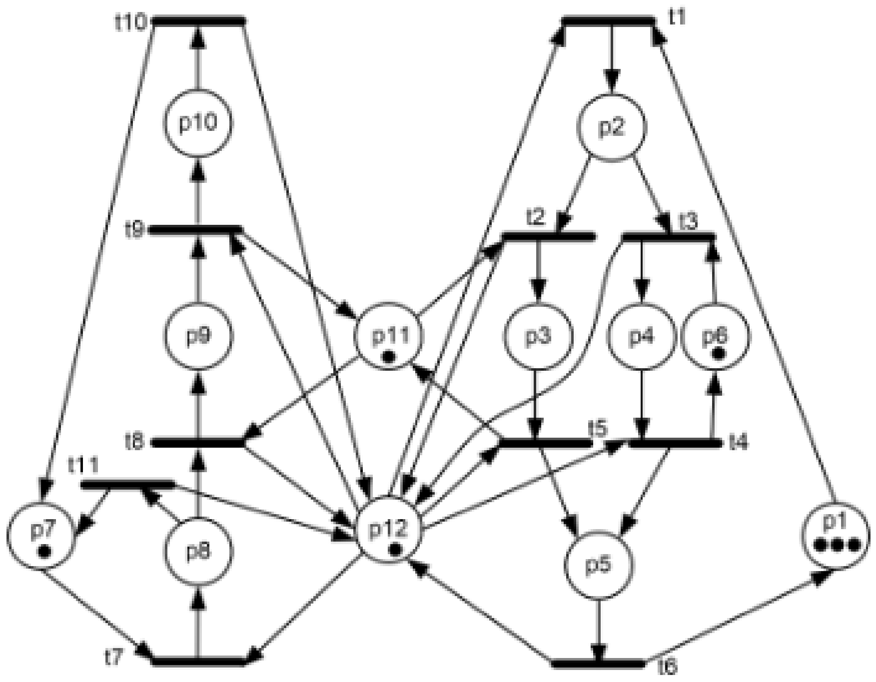
Proposition 2. Let *C* be a circuit in the resource flow graph of a conflict-free S^3PR *N* with an acceptable initial marking M_0. If it is possible to assign to every arc in *C* a weight (a natural number) such that:

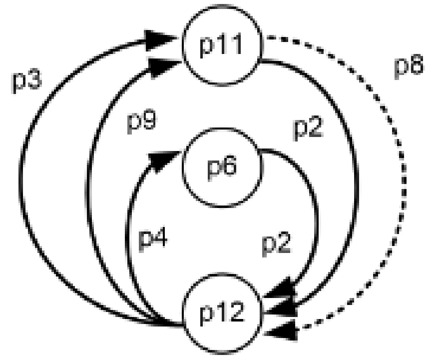
(1) for every resource place *r* involved in *C*, the sum of the weights of the incoming arcs is equal to $M_0(r)$, and

(2) the sum of the weights of the arcs, labeled with the activity places belonging to the same process N_i, is not more than $M_0(p_{0i})$, where p_{0i} is the idle place of N_i,

then there is a possible marking for *N* and M_0, being a deadlock.

Now consider the nets with conflicts

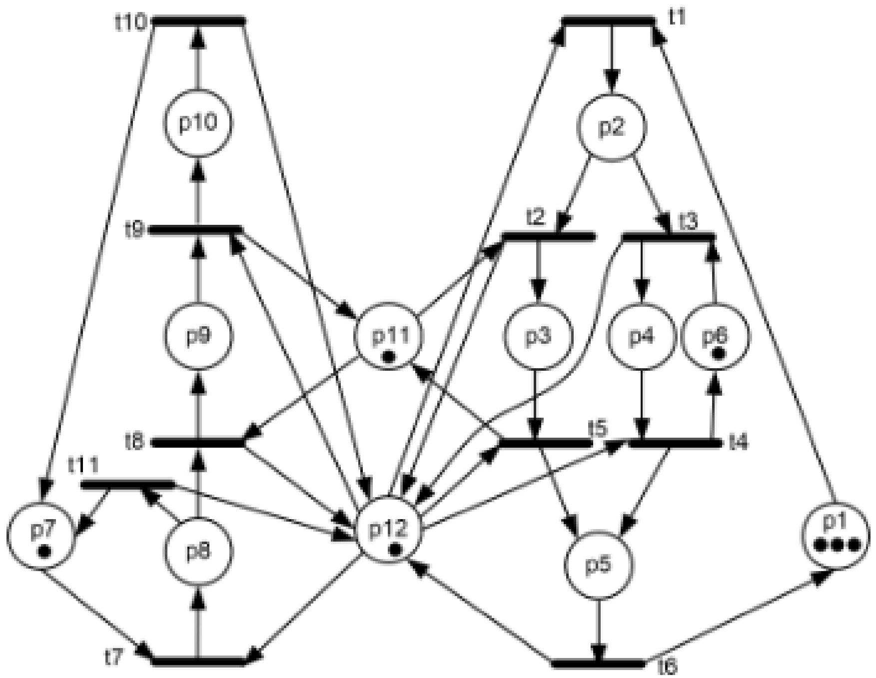


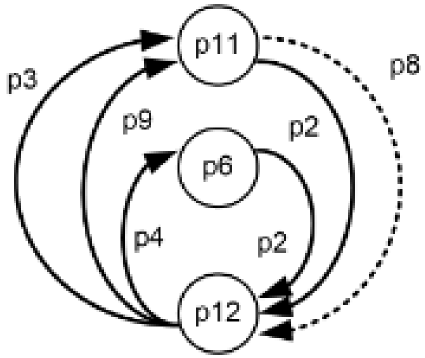


p2, p9 – no deadlock

p3, p8 – no deadlock

Now consider the nets with conflicts





p2, p9 – no deadlock
(both arcs labeled with p2 should be involved)

p3, p8 – no deadlock
(p8 can never be deadlocked)

Conclusions on the case of S^3PRs with conflicts:

- If in a deadlock-prone circuit $C = (V, A)$ there is an arc $a_1 \in A$ labeled with a place p, and in the resource flow graph there is another arc a_2 also labeled with p, then $a_2 \in A$
- If there is an activity place p and a transition t such that ${}^\bullet t = \{p\}$, then no arc of the resource flow graph labeled with p belongs to a deadlock-prone circuit.

Deadlock-prone circuit: a general definition

A ***deadlock-prone circuit*** of an S^3PR $N = (P_A \cup P^0 \cup P_R, T, F)$ with an acceptable initial marking M_0 is a circuit $C = (V_C, A_C)$ in its resource flow graph $G = (V, A)$, such that:

- if there are arcs a_i and a_j in A labeled with the same activity place, and $a_i \in A_C$, then $a_j \in A_C$;
- if an arc a is labeled with place p such that $\exists t \in T : {}^\bullet t = \{p\}$, then $a \notin A_C$;
- Let $p_r \in V_C$ and $s(p_r)$ be the set of activity places labeling the incoming arcs of p_r in C. There exists a function $f(p)$ that assigns a positive integer to every activity place p labeling an arc in C such that:
  - $\forall r \in V_c: \sum_{p \in s(r)} f(p) = M_o(r)$;
  - For every S^2PR $N_i = (P_{Ai} \cup \{p_i^0\} \cup P_{Ri}, T_i, F_i)$ in N, let $P_{A_i}^C$ be the subset of P_{Ai} such that the places belonging to $P_{A_i}^C$ label some arcs in C. Then $\sum_{p \in P_{A_i}^C} f(p) \le M_0(p_i^0)$.

The final result

Proposition 3. If in the resource flow graph of an S^3PR $N = (P_A \cup P^0 \cup P_R, T, F)$ with an acceptable initial marking M_0 there exists a deadlock-prone circuit $C = (V_C , A_C)$*, then there is a possible marking M_d of N being a deadlock.

* satisfying the definition presented above

Conclusion and further work

- The notion of a deadlock-prone circuit is introduced for parallel systems modeled by S^3PR nets, which enables deadlock detection in more general cases than the well-known cycles in resource flow graphs allow
- Future work is going to be focused on the reachability issues and on generalization of the results for wider classes of the nets (e.g., S^4R)