

# Toward Boolean Functions in Post -Quantum Cryptography

This talk explores the role of Boolean functions in post-quantum cryptography, with a focus on their use in practical implementations. We focus on post-quantum cryptography and side-channel attacks focusing to KYBER and Dilithium algorithms. The presentation then examines the arithmetic operations underlying these schemes, emphasizing their Boolean structure, masking techniques, and transformations between arithmetic and Boolean domains. We discuss an approach to implementing arithmetic operations via superposition of low-bit-width partial products

Danila Gorodecky (Gorodetsky)

*INESC-ID, Instituto Superior Tecnico, Universidade de Lisboa*



## Two parties can establish a secure channel over a fully public network, without sharing a secret in advance

RSA (Rivest–Shamir–Adleman) security relies on integer factorization of  $A \cdot B = N(\text{mod } P)$ , where  $A$ ,  $B$ ,  $N$  and  $P$  are up to 4096-bit integers

ECC (Elliptic Curve Cryptography) security relies on  $A \cdot k = Q(\text{mod } P)$ ,  $A$ ,  $k$ ,  $Q$  and  $P$  are up to 512-bit integers

**Implementation:** HTTPS/TLS, cryptocurrency (Bitcoin, blockchain), Signal, WhatsApp, Telegram, VPNs, digital signatures, e-passports, SSH, etc.

**P. Shor (1994) showed that a sufficiently large quantum computer can solve both factorization (RSA) and the discrete logarithm (ECC) in polynomial time**

Today's quantum processors operate with hundreds to thousands of physical qubits.

Cryptographically relevant attacks require millions of error-corrected logical qubits (estimated 15–20 years away).

**Post-quantum cryptography is the design of cryptographic algorithms that remain secure against both classical and quantum computers, based on mathematical problems for which no efficient quantum algorithm is known**

An algorithm can be mathematically unbreakable yet physically exploitable

**What leaks:**

- **Power consumption** — an attacker connects an oscilloscope to a smart card. They send  $N$  known plaintexts and record  $N$  power traces
- **Timing** — execution time depends on secret-dependent branches and memory accesses
- **Electromagnetic emission** — radiation from the chip carries data-dependent patterns
- **Cache behavior** — memory access patterns reveal key-dependent indices

**PQC protects against quantum attacks, but not against side-channel leakage**

recovering a shared secret from a ciphertext using a secret key

In **decapsulation**, the device processes a secret key together with attacker-controlled input, creating an scenario for power analysis attacks, making unprotected implementations as vulnerable as classical schemes like RSA. A 128-bit key is recovered in minutes from a few thousand traces.

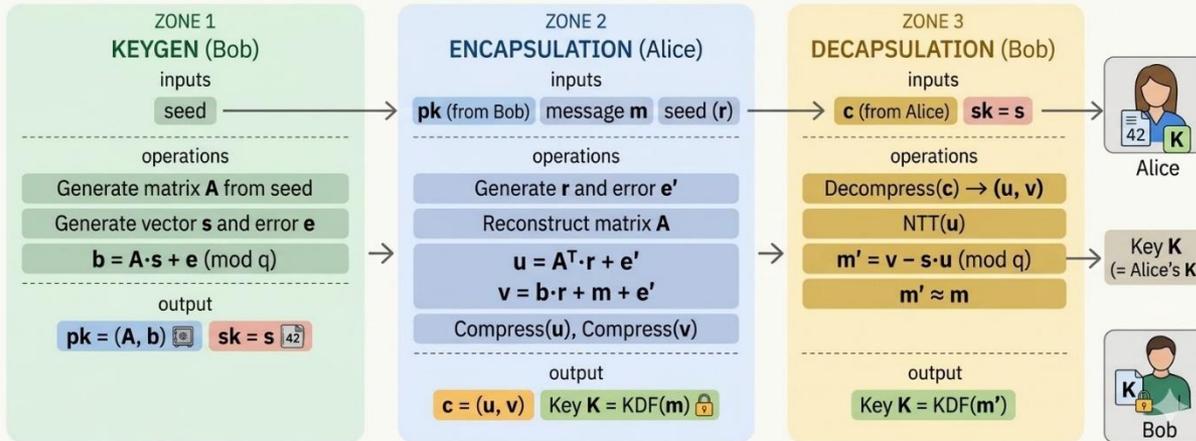
**Boolean masking** — splitting every secret value into  $d$  random shares before any computation. Cost scales as  $O(d)$  for XOR ( $\oplus$ ) operations,  $O(d^2)$  for AND operations.

Minimizing AND-gate count in arithmetic operations is the central problem! 3

# NIST Post-Quantum Cryptography (PQC): KYBER and Dilithium

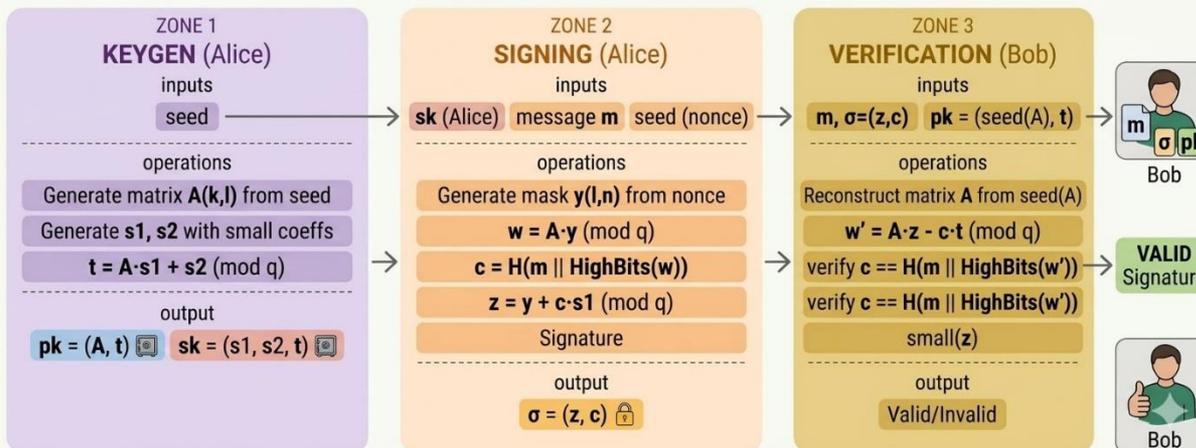
RSA and ECC are mathematically broken by quantum computing.  
The transition to PQC is not optional, but it is a matter of timing

CRYSTALS-Kyber (ML-KEM-512)  $q = 3329 \cdot n = 256$  coefficients  $\times$  12 bit



Operation	Comments
$A \cdot c \pmod{q}$	Matrix $\times$ vector: multiply 256-coeff polynomials entry by entry based on NTT (Number Theoretic Transform) — the fast algorithm for multiplying polynomials. Without it: $256^2 = 65\,536$ multiplications, with NTT: $256 \cdot \log_2(256) = 2\,048$ . Cost: $O(n \log n)$ instead of $O(n^2)$
Compress $[x \cdot 2^d / q]$	Round a coefficient from 12 bits down to $d$ bits. Discards precision to shrink the ciphertext. Like rounding $3.14159 \rightarrow 3.1$
Decompress $[x \cdot q / 2^d]$	Reconstruct approximate 12-bit value from $d$ bits. Small error is tolerable — the noise $e$ already introduced imprecision
HighBits $[w/a]$	Keep only the coarse part of a coefficient. Like keeping only "hours" from a time value, ignoring minutes
LowBits $w \pmod{a}$	Keep only the fine remainder. Like keeping only "minutes". Together High+Low = full value
B2A $\leftrightarrow$ A2B $x_0 \oplus x_1 \leftrightarrow x_0 + x_1$	Convert secret shares from/to XOR-form to/from addition-form. Needed when switching between bitwise and arithmetic operations on a masked secret

CRYSTALS-Dilithium (ML-DSA-65)  $q = 8380417 \cdot k=6, l=5, n=256$  coeff  $\times$  12 bit



## 1 The idea: never process the secret directly

$$\text{secret } x = x_0 \text{ (random)} \oplus x_1 \text{ (random)} \oplus x_2 \text{ (random)}$$

The secret  $x$  is split into  $d$  random shares. Each share alone looks completely random. It reveals nothing about  $x$ . The device always works with shares, never with  $x$  itself. An attacker observing power consumption sees only random values.

## 2 Concrete example ( $d = 2$ shares, 4-bit numbers)

secret:  $x = 1011_2 = 11$

random  $r$ :  $r = 0110_2 = 6$

share 0:  $x_0 = x \oplus r = 1011 \oplus 0110 = 1101$  (looks random)

share 1:  $x_1 = r = 0110$  (looks random)

restore:  $x_0 \oplus x_1 = 1101 \oplus 0110 = 1011 = 11 \checkmark$

## 3 Cost of operations on masked values

### XOR $\oplus$ cheap

$$z = x \oplus y$$

$$z_i = x_i \oplus y_i$$

Each share computed independently — **no interaction between shares**.

Total cost:  $d$  operations  $\rightarrow O(d)$

### AND $\cdot$ expensive

$$z = x \cdot y$$

$$z_i = x_i \cdot y_i \oplus \sum r_{ij}$$

Shares of  $x$  and  $y$  **must interact** — each pair  $(i,j)$  needs a fresh random bit  $r_{ij}$ .

Total cost:  $d^2$  operations  $\rightarrow O(d^2)$

## 4 Why AND is expensive — the cross-term problem

### XOR of two masked values

$$(x_0 \oplus x_1) \oplus (y_0 \oplus y_1) = (x_0 \oplus y_0) \oplus (x_1 \oplus y_1)$$

Each share only needs its own pair. No cross-terms. Safe to compute separately.

### AND of two masked values

$$(x_0 \oplus x_1) \cdot (y_0 \oplus y_1) = x_0 y_0 \oplus x_0 y_1 \oplus x_1 y_0 \oplus x_1 y_1$$

Cross-terms  $x_0 y_1$  and  $x_1 y_0$  appear — they correlate with both  $x$  and  $y$ . Each must be randomized with a fresh  $r_{ij}$ .

$$\text{Cost}(d) = X \cdot d + A \cdot d^2$$

$X$  = number of XOR gates ·  $A$  = number of AND gates ·  $d$  = masking order

**Minimizing  $A$  directly reduces the cost of any masked implementation.**

For  $d = 4$ : cutting  $A$  in half saves  $8\times$  more operations than cutting  $X$  in half.

Optimized operation	Kyber	Dilithium	AES
<b>modular multiplication</b> $(A \cdot B) \bmod q$	NTT butterfly: $\omega \cdot b \pmod{3329}$ . ~1024 multiplications per NTT, multiple NTT calls per decapsulation	NTT butterfly: $\omega \cdot b \pmod{8380417}$ . Same structure as ML-KEM. 23-bit modulus = larger Boolean functions, greater optimization potential	—
<b>modular reduction</b> $A \bmod q$	Internal to NTT, Compress, B2A/A2B mask conversions	Internal to NTT, rejection sampling.	Reduction mod $m(x)$ in S-box tower-field decomposition
<b>multiplication by constant (C)</b> $(C \cdot A) \pmod{q}$	NTT twiddle factors: $\omega^i \cdot b \pmod{3329}$ . Constants known at design time	NTT twiddle factors: $\omega^i \cdot b \pmod{8380417}$ . Constants known at design time	—
<b>division by Constant (d)</b> $A / d = \{Q, R\}$	Compress: $\lceil 2^d/q \cdot x \rceil$ Division by $q = 3329$	—	—
<b>B2A and A2B mask conversion</b>	Compress: A2B after NTT. Decompress: B2A before NTT. CCA comparison: A2B. ~256 conversions per polynomial	Similar conversions for rejection sampling and signature verification	—

## high-level representation

“well-suited”  
algebraic formula

splitting input and  
output operands into  
sub-words of smaller  
bit-width

representation of the  
formula as a  
superposition of  
“simple” arithmetic  
functions

## logic design

Boolean  
representations

properties of  
Boolean functions

Boolean  
representations

Boolean  
minimizations

**Efficient representation of  
arithmetic operation**

- Modular operations (for an arbitrary  $P$ )
  - constant multiplication  $((A \cdot \text{const})(\text{mod } P) = R)^*$
  - modular multiplication  $((A \cdot B)(\text{mod } P) = R)^*, **$
  - modular addition  $((A + B)(\text{mod } P) = S)$
  - modular reduction  $(A(\text{mod } P) = R)^*$
  - Modular division  $((A \cdot x)(\text{mod } P) \equiv 1)$
  - Residue Number System (RNS)
- Standard operations
  - constant multiplication  $(A \cdot \text{const} = R)^{****}$
  - multiplication  $(A \cdot B = R)^{*****}$
  - addition  $(A + B = S)$
  - division by constant  $(A = \text{constant} \cdot Q + \text{residue})^{**}$
- Floating-point calculations
  - multiplication  $(A \cdot B = R)$
  - addition  $(A + B = S)$
  - normalization

\*D. Gorodecky, L. Sousa, "Modular Arithmetic Based on Boolean Functions: A Divide and Conquer Approach" // IEEE Access, Vol. 13, Oct., 2025, pp. 186383-186396.

\*\*D. Gorodecky, L. Sousa, "Scalable architecture of constant division on FPGA" // Proceedings of the 30th IEEE Symposium on Computer Arithmetic, Sep. 4-6, 2023, Portland, Oregon, USA.

\*\*\*D. Gorodecky, L.Sousa, "Two-Operand Modular Multiplication to Small Bit Ranges" // Advanced Boolean Techniques, Springer Nature Switzerland, 2023, p. 111-122.

\*\*\*\*D. Gorodecky, P. Bibilo, "Constant Multiplication Based on Boolean Minimization" // Proceedings of the 14th International Workshop on Boolean Problems, Bremen, Germany, Sept. 24-25, 2020.

\*\*\*\*\*D. Gorodecky "Design of Multipliers Using Fourier Transformations" // Further Improvements in the Boolean Domain, Cambridge Scholars Publishing, UK, 2018, Section 3.4, pp. 240-252.

split operation into partial products of smaller sub-words

## Arithmetic Decomposition

$$F = A_1 \cdot B_1 \cdot C_1 + A_2 \cdot B_2 \cdot C_2 + \dots + A_w \cdot B_w \cdot C_w$$

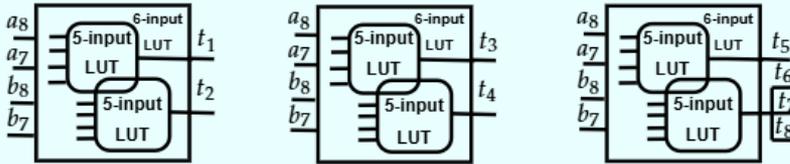
represent each partial product as a Boolean function  $g_i(X)$

## Boolean Mapping

$$A_1 \cdot B_1 \cdot C_1 = g_1(X_1), A_2 \cdot B_2 \cdot C_2 = g_2(X_2), \dots, A_w \cdot B_w \cdot C_w = g_w(X_w)$$

map minimized functions to FPGA LUTs or ASIC standard cells

## LUT/Library Mapping



## Boolean Decompos./Minimiz.

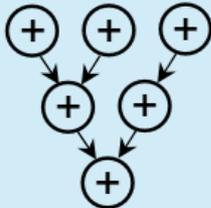
$$F = g_1(h_1(X_1), \dots, h_v(X_v)) \circ g_2(h_1(X_1), \dots, h_v(X_v)) \circ \dots \circ g_w(h_1(X_1), \dots, h_v(X_v))$$

"o" ∈ {+, -, ·}

minimize Boolean functions using Reed-Muller, Karnaugh, or technology-dependent methods

## Adders Tree

ballanced parallel additions



combine partial results using balanced parallel addition

## Result Integration

concatenation

$R - P$  if  $R \geq P$   
modular correction  
when required

concatenate sub-results and apply modular correction if  $R \geq P$

$$(A \cdot B) \pmod{3329} = R$$

$$A = (a_{12} a_{11} \dots a_1)$$

$$B = (b_{12} b_{11} \dots b_1)$$

$$A_1 = (a_4 a_3 a_2 a_1)$$

$$B_1 = (b_4 b_3 b_2 b_1)$$

$$A_2 = (a_8 a_7 a_6 a_5)$$

$$B_2 = (b_8 b_7 b_6 b_5)$$

$$A_3 = (a_{12} a_{11} a_{10} a_9)$$

$$B_3 = (b_{12} b_{11} b_{10} b_9)$$

$$A \cdot B = R \pmod{3329} =$$

$$A_1 \cdot B_1 + A_1 \cdot B_2 \cdot 2^4 + A_1 \cdot B_3 \cdot 2^8 +$$

$$A_2 \cdot B_1 \cdot 2^4 + A_2 \cdot B_2 \cdot 2^8 + A_2 \cdot B_2 \cdot 2^{12} +$$

$$A_3 \cdot B_1 \cdot 2^8 + A_3 \cdot B_2 \cdot 2^{12} + A_3 \cdot B_3 \cdot 2^{16} = A_1 \cdot B_1 + A_1 \cdot B_2 \cdot 2^4 + A_1 \cdot B_3 \cdot 2^8 +$$

$$A_2 \cdot B_1 \cdot 2^4 + A_2 \cdot B_2 \cdot 2^8 + A_2 \cdot B_2 \cdot 767 +$$

$$A_3 \cdot B_1 \cdot 2^8 + A_3 \cdot B_2 \cdot 767 + A_3 \cdot B_3 \cdot 2285$$

High – Level Representation

Building blocks:  
8 input Boolean functions

$$A_1 \cdot B_1$$

$$A_1 \cdot B_2 \cdot 2^4$$

$$A_1 \cdot B_3 \cdot 2^8$$

$$A_2 \cdot B_2 \cdot 767$$

$$A_3 \cdot B_3 \cdot 2285$$

# $A \cdot B \pmod{241} = R^*$

$$A = (a_8 a_7 \dots a_1) \quad B = (b_8 b_7 \dots b_1)$$

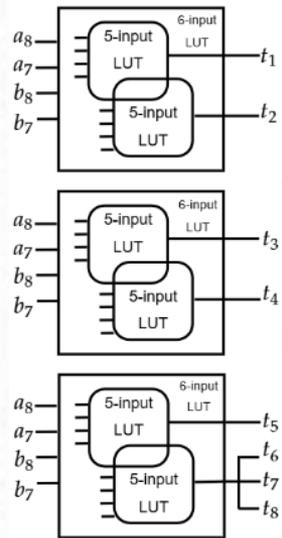
$$(A \cdot B) \pmod{241} =$$

$$((a_3 a_2 a_1) \cdot (b_3 b_2 b_1) + (a_3 a_2 a_1) \cdot (b_6 b_5 b_4) \cdot 2^3 + (a_3 a_2 a_1) \cdot (b_8 b_7) \cdot 2^6 +$$

$$(a_6 a_5 a_4) \cdot (b_3 b_2 b_1) \cdot 2^3 + (a_6 a_5 a_4) \cdot (b_6 b_5 b_4) \cdot 2^6 + (a_6 a_5 a_4) \cdot (b_8 b_7) \cdot 30 +$$

$$(a_8 a_7) \cdot (b_3 b_2 b_1) \cdot 2^6 + (a_8 a_7) \cdot (b_6 b_5 b_4) \cdot 30 + (a_8 a_7) \cdot (b_8 b_7) \cdot 240 \pmod{241} =$$

$$R = (r_8 r_7 \dots r_1)$$



$$t_1 = a_8 \cdot (b_8 \cdot \overline{b_7} \vee \overline{a_7} \cdot b_7) \vee a_7 \cdot b_8 \cdot \overline{b_7} =$$

$$\overline{b_7} \oplus \overline{b_8} \cdot \overline{b_7} \oplus \overline{a_7} \oplus \overline{a_7} \cdot \overline{b_7} \oplus$$

$$a_8 \cdot a_7 \oplus a_8 \cdot a_7 \cdot b_8 \cdot \overline{b_7},$$

$$t_2 = a_8 \cdot b_7 \cdot (\overline{a_7} \vee \overline{b_8}) \vee a_7 \cdot b_8 \cdot (\overline{a_8} \vee \overline{b_7}) =$$

$$a_7 \cdot b_8 \oplus a_8 \cdot b_7,$$

$$t_3 = a_8 \cdot (\overline{b_8} \cdot b_7 \vee \overline{a_7} \cdot b_8 \cdot \overline{b_7}) \vee \overline{a_8} \cdot a_7 \cdot b_8 =$$

$$a_7 \cdot b_8 \oplus a_8 \cdot b_7 \oplus a_8 \cdot b_8 \oplus a_8 \cdot b_8 \cdot b_7 \cdot b_8,$$

$$t_4 = a_8 \cdot (b_8 \vee b_7) \vee a_7 \cdot b_8 =$$

$$a_7 \cdot b_8 \oplus a_8 \cdot b_7 \oplus a_8 \cdot b_8 \oplus a_8 \cdot b_8 \cdot b_7 \oplus a_8 \cdot a_7 \cdot b_8,$$

$$t_5 = \overline{a_8} \cdot a_7 \cdot \overline{b_8} \cdot b_7 =$$

$$a_7 \cdot b_7 \oplus a_7 \cdot b_8 \cdot b_7 \oplus a_8 \cdot a_7 \cdot b_7 \oplus a_8 \cdot a_7 \cdot b_8 \cdot b_7,$$

$$t_6 = t_7 = t_8 = (a_8 \vee a_7) \cdot (b_8 \vee b_7) =$$

$$1 \oplus \overline{b_8} \cdot \overline{b_7} \oplus \overline{a_8} \cdot \overline{a_7} \oplus \overline{a_8} \cdot \overline{a_7} \cdot \overline{b_8} \cdot \overline{b_7}$$

$$((a_8 a_7) \cdot (b_8 b_7) \cdot 240) \pmod{241} = T(t_8 t_7 \dots t_1)$$

decimal	$(a_8 a_7) \cdot (b_8 b_7) \cdot 240$	$a_8$	$a_7$	$b_8$	$b_7$	$t_8$	$t_7$	$t_6$	$t_5$	$t_4$	$t_3$	$t_2$	$t_1$	decimal $T$
0 · 0 · 240	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0 · 1 · 240	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0 · 2 · 240	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0 · 3 · 240	0	0	1	1	0	0	0	0	0	0	0	0	0	0
1 · 0 · 240	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1 · 1 · 240	0	1	0	1	1	1	1	0	0	0	0	0	0	240
1 · 2 · 240	0	1	1	0	1	1	1	0	1	1	1	1	1	239
1 · 3 · 240	0	1	1	1	1	1	1	0	1	1	1	0	1	238
2 · 0 · 240	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2 · 1 · 240	1	0	0	1	1	1	1	0	1	1	1	1	1	239
2 · 2 · 240	1	0	1	0	1	1	1	0	1	1	0	1	1	237
2 · 3 · 240	1	0	1	1	1	1	1	0	1	0	1	0	1	235
3 · 0 · 240	1	1	0	0	0	0	0	0	0	0	0	0	0	0
3 · 1 · 240	1	1	0	1	1	1	1	0	1	1	0	1	1	238
3 · 2 · 240	1	1	1	0	1	1	1	0	1	0	1	0	1	235
3 · 3 · 240	1	1	1	1	1	1	1	0	1	0	0	0	0	232

$$X(\text{mod } 5) = S^*$$

VHDL

$$X = (x_5 x_4 x_3 x_2 x_1)$$

5-bit number

$$S = (S_3 S_2 S_1)$$

3-bit number

Reed-Muller form

$S(1) <= x(1) \text{ xor } (x(1) \text{ and } x(3)) \text{ xor } (x(2) \text{ and } x(3)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3)) \text{ xor } x(4) \text{ xor } (x(2) \text{ and } x(4)) \text{ xor } (x(3) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(3) \text{ and } x(4)) \text{ xor } x(5) \text{ xor } (x(3) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(2) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(3) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3) \text{ and } x(4) \text{ and } x(5));$

$$S_1 = x_1 \oplus x_1 \cdot x_3 \oplus x_2 \cdot x_3 \oplus x_1 \cdot x_2 \cdot x_3 \oplus x_4 \oplus x_2 \cdot x_4 \oplus x_3 \cdot x_4 \oplus x_1 \cdot x_3 \cdot x_4 \oplus x_5 \oplus x_3 \cdot x_5 \oplus x_1 \cdot x_3 \cdot x_5 \oplus x_2 \cdot x_3 \cdot x_5 \oplus x_1 \cdot x_2 \cdot x_3 \cdot x_5 \oplus x_1 \cdot x_4 \cdot x_5 \oplus x_1 \cdot x_2 \cdot x_4 \cdot x_5 \oplus x_3 \cdot x_4 \cdot x_5 \oplus x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5$$

$S(2) <= x(2) \text{ xor } (x(2) \text{ and } x(3)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3)) \text{ xor } x(4) \text{ xor } (x(1) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(3) \text{ and } x(4)) \text{ xor } (x(2) \text{ and } x(3) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(2) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(4) \text{ and } x(5)) \text{ xor } (x(2) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(3) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(3) \text{ and } x(4) \text{ and } x(5));$

$$S_2 = x_2 \oplus x_2 \cdot x_3 \oplus x_1 \cdot x_2 \cdot x_3 \oplus x_4 \oplus x_1 \cdot x_4 \oplus x_1 \cdot x_2 \cdot x_4 \oplus x_1 \cdot x_3 \cdot x_4 \oplus x_2 \cdot x_3 \cdot x_4 \oplus x_1 \cdot x_5 \oplus x_1 \cdot x_3 \cdot x_5 \oplus x_2 \cdot x_3 \cdot x_5 \oplus x_1 \cdot x_2 \cdot x_3 \cdot x_5 \oplus x_4 \cdot x_5 \oplus x_2 \cdot x_4 \cdot x_5 \oplus x_3 \cdot x_4 \cdot x_5 \oplus x_1 \cdot x_3 \cdot x_4 \cdot x_5$$

$S(3) <= x(3) \text{ xor } (x(1) \text{ and } x(3)) \text{ xor } (x(2) \text{ and } x(3)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3)) \text{ xor } (x(1) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(4)) \text{ xor } (x(3) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(3) \text{ and } x(4)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(5)) \text{ xor } (x(3) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(2) \text{ and } x(3) \text{ and } x(5)) \text{ xor } (x(4) \text{ and } x(5)) \text{ xor } (x(2) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(1) \text{ and } x(2) \text{ and } x(4) \text{ and } x(5)) \text{ xor } (x(2) \text{ and } x(3) \text{ and } x(4) \text{ and } x(5));$

$$S_3 = x_3 \oplus x_1 \cdot x_3 \oplus x_2 \cdot x_3 \oplus x_1 \cdot x_2 \cdot x_3 \oplus x_1 \cdot x_4 \oplus x_1 \cdot x_2 \cdot x_4 \oplus x_3 \cdot x_4 \oplus x_1 \cdot x_2 \cdot x_3 \cdot x_4 \oplus x_1 \cdot x_2 \cdot x_5 \oplus x_3 \cdot x_5 \oplus x_1 \cdot x_3 \cdot x_5 \oplus x_2 \cdot x_3 \cdot x_5 \oplus x_4 \cdot x_5 \oplus x_2 \cdot x_4 \cdot x_5 \oplus x_1 \cdot x_2 \cdot x_4 \cdot x_5 \oplus x_2 \cdot x_3 \cdot x_4 \cdot x_5$$

The (minimized) Reed-Muller representation expresses every output bit using **ONLY** AND and XOR operations

- In Boolean masking, XOR is free (linear, applied to each share independently)
- AND is expensive (requires ISW gadget, cost scales as  $d^2$  per gate)
- The number of AND gates in the Reed-Muller form directly determines the cost of the masked implementation
- Optimization reduces the number of AND gates → directly reduces masking cost

\*D. Gorodecky, "Reed-Muller Realization of  $X \pmod{P}$ ", 2015, arXiv:1504.04773 [cs.DM].  
 \*\*D. Gorodecky, "Reed-Muller Representation in Arithmetic Operations" // Reed-Muller Workshop 2019, May 24, 2019, Fredericton, New Brunswick, Canada, pp. 53-58.  
 \*\*\*D. Gorodecky, " $X \pmod{2^k - 1}$  calculation based on Reed-Muller expansions of symmetric Boolean functions" // Reed-Muller Workshop 2023, May 24, 2023, Matsue, Shimane Prefecture, Japan

**Vivado, Synopsys, Cadence** are the main commercial Electronic Design Automation (EDA) tools

## FPGA (Vivado)\*

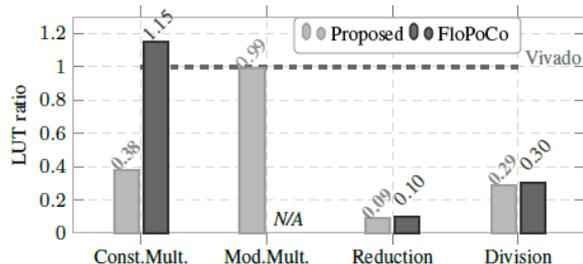


Fig. 1: LUT utilization comparison (normalized to Vivado = 1.0, geometric mean). Lower is better.

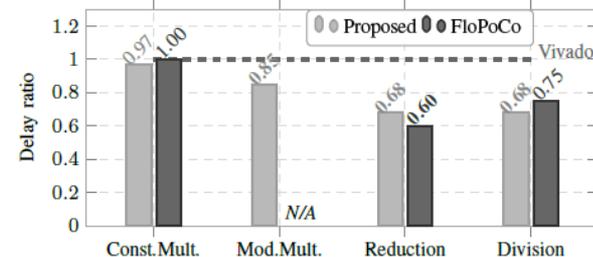


Fig. 2: Critical path delay comparison (normalized to Vivado = 1.0, geometric mean). Lower is better.

## ASIC (Synopsys)

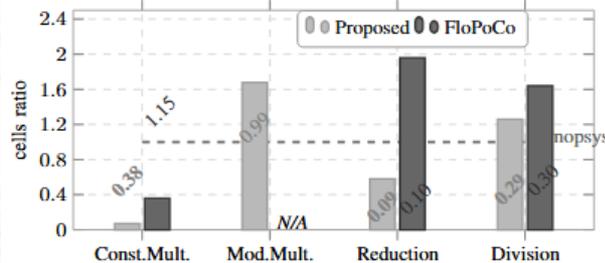


Fig. 1: cells utilization comparison (normalized to Synopsys = 1.0, geometric mean). Lower is better.

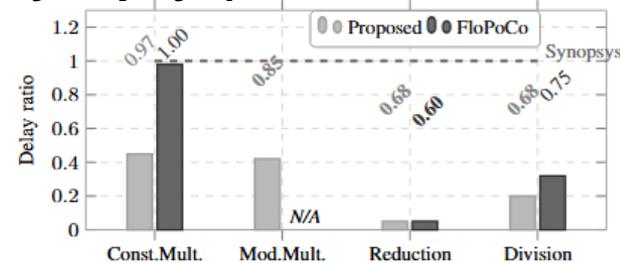


Fig. 2: Critical path delay comparison (normalized to Synopsys = 1.0, geometric mean). Lower is better.

## ASIC (Cadence)

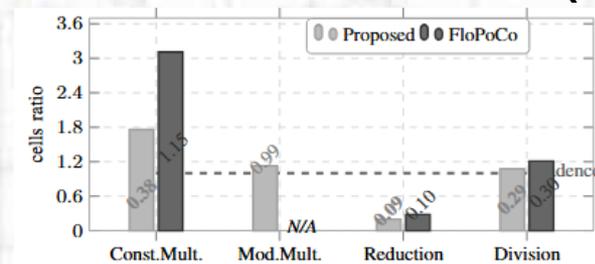


Fig. 1: cells utilization comparison (normalized to Cadence = 1.0, geometric mean). Lower is better.

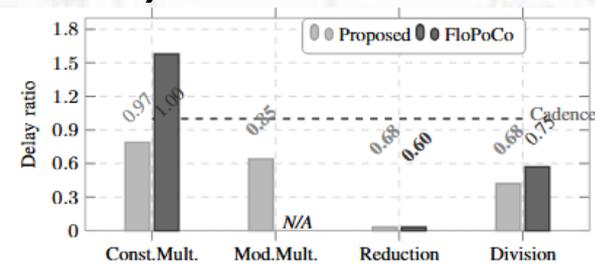


Fig. 2: Critical path delay comparison (normalized to Cadence = 1.0, geometric mean). Lower is better.

**RISC-V and ARM Cortex-M** processors are built from the same **ASIC** cell libraries

Blocks of ASIC standard cells (AND, OR, XOR, MUX, INV, etc) correspond to basic processor operations.

Optimizing gate-level circuits for ASIC libraries simultaneously optimizes for software implementations on RISC-V and ARM Cortex-M